A four colour problem

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The final question in the 2020 Australian Mathematics Competition (Junior Years 7 - 8) asked students to determine how many different ways sixteen squares of four different colours can be arranged so that no two squares of the same colour share a side or corner. Figure 1 shows the example that was provided to illustrate an arrangement that is not acceptable as two yellow squares share a corner.

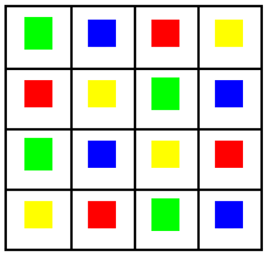

Figure 1: An unacceptable arrangement - two yellow squares share a corner

The first step in finding a solution is to create a base arrangement consisting of four identical 2×2 squares (Figure 2). There are 4×3×2×1 = 24 ways of arranging the four colours in the 2×2 square so there are 24 different variations of the base arangement.

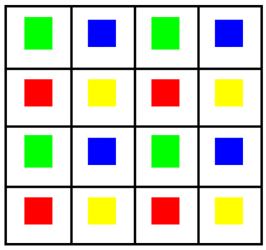

Figure 2: The base arrangement

We can then introduce variability by first swapping the order in the third column, then the fourth column (after resetting the third column) and finally both the third and fourth columns:

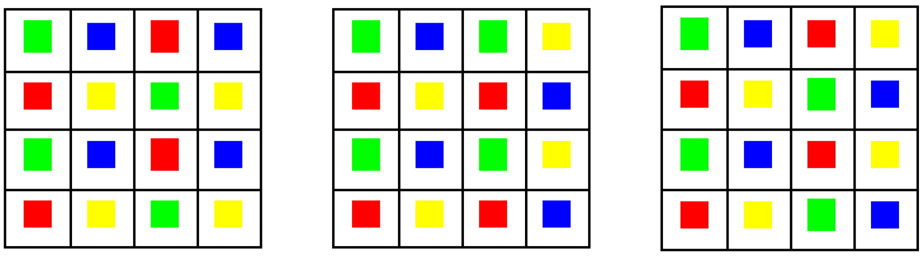

Figure 3: Changing the third and fourth columns

Returning to the base arrangement we now repeat the process by swapping first the third row, then the fourth row and finally both third and fourth rows:

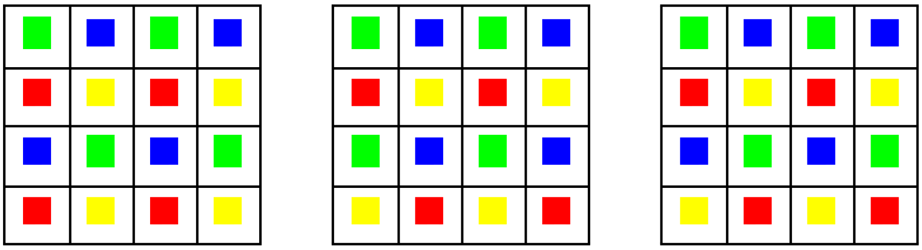

Figure 4: Changing the third and fourth rows

Hence there are six variations to the base arrangement, each with 24 arrangements (there are 24 arrangemnts in the top left 2×2 square and the remaining 12 squares are related to the arrangement of these four squares).

Therefore in total there are 7 × 24 = 168 different arrangements of the 16 squares such that no two squares of the same colour share a side or corner.