Triangular pattern with the numbers 1 to 6

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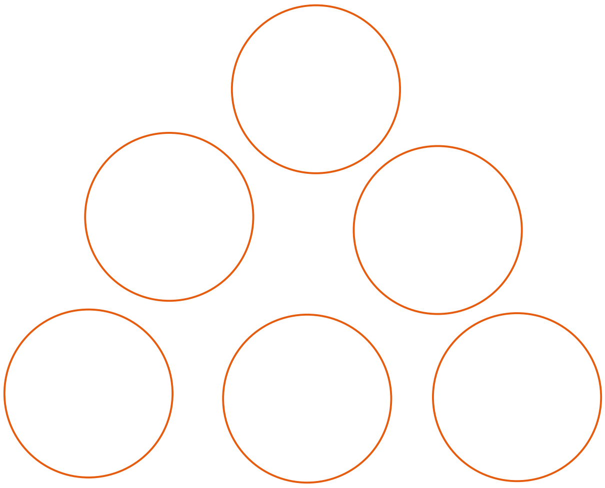
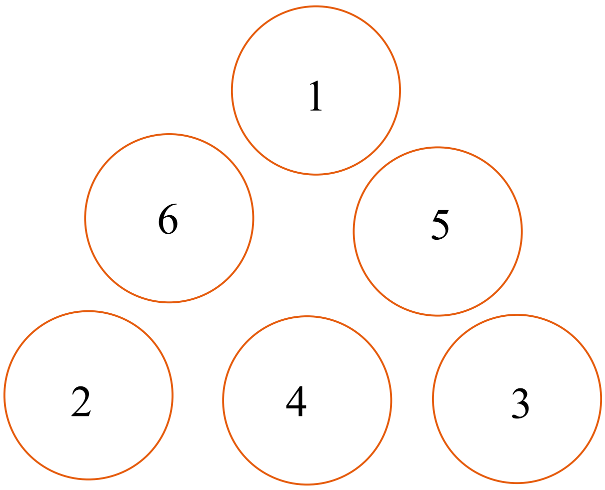
The object of this problem is to place the numbers 1 to 6 in the circles so that the sum of the three numbers in each edge are equal.

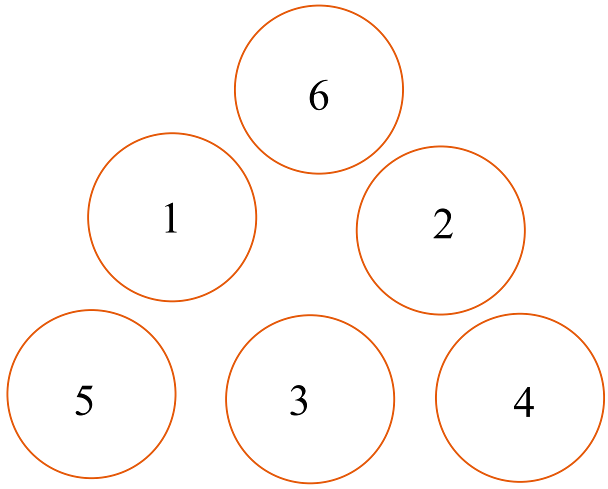


Here are four examples:

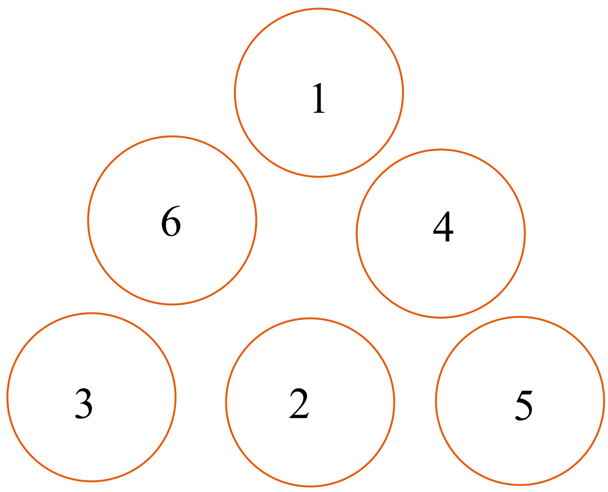
Here the sum is 9 on each edge



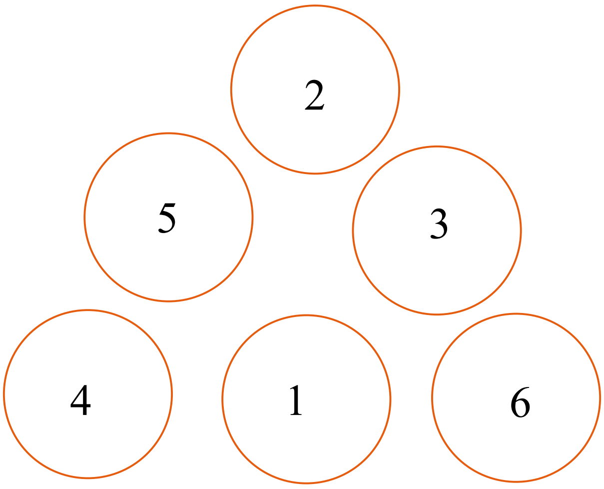
In this example the sum is 12



Now the sum is 10



Finally the sum is 11



How is this determined? Do you see a pattern in each example?

In the first example the numbers in the corner circle are the lowest thee (1, 2, 3).

In the second example the corner numbers are the highest three (4, 5, 6)

In the third example the corner numbers are odd (1, 3, 5) while in the last example they are even (2, 4, 6).

As the pattern has three sides and must have equal sum along each side the sum of the numbers in the pattern must be a multiple of 3.

The sum of the numbers 1, 2, ..,6 is 21 which is divisible by 3. The sum of the numbers in the pattern will be greater than 21 because the numbers in the corner circles are counted twice. Therefore a condition for a pattern to be feasable requires the sum of the corner numbers to be divisible by 3. In the four examples shown above we have 1 + 2 + 3 = 6 = 2 x 3, 4 + 5 + 6 = 15 = 5 x 3, 1 + 3 + 5 = 9 = 3 x 3 and 2 + 4 + 6 = 12 = 4 x 3).

However this is not a sufficient condition since 1 + 2 + 6 = 9 = 3 x 3 giving a pattern total of 21 + 9 = 30 and thus a sum of 10 along each side. This is impossble with the remaining numbers (3, 4, 5).

Reversing this, choosing the corner numbers as (3, 4, 5) gives a sum of 11 along each side but this is impossible with the remaining numbers (1, 2, 6).

Similarly the choices (1, 5, 6) and (2, 3, 4) are possible corner numbers but result in impossible patterns with the remaining numbers available.