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HSC Extension 1 Mathematics Revision Questions - Set 3

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1. Find $\int \cos x \sin^6 x \, dx$
2. Find
 - (i) $\lim_{x \rightarrow 0} \frac{\sin 5x}{3x}$
 - (ii) $\lim_{x \rightarrow 0} \frac{2x}{\sin 4x}$
3. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 6x \, dx$
4. Solve $\cos 2\theta = \sin \theta$
5. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos \theta}{2 - \sin \theta} \, d\theta$
6. Solve $5 \cos \theta - 4 \sin \theta = 1$
7. Find $\frac{d}{dx} \tan[\cos(\ln x)]$
8. Differentiate $\ln(\cot x)$
9. Differentiate $x \cos 2x$ and hence evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} x \sin 2x \, dx$
10. Find the exact value of $\tan \frac{\pi}{8}$
11. Differentiate $\ln\left(\frac{x\sqrt{x+1}}{x^2-2}\right)$
12. Find any stationary and inflection points on the curve $y = x^2 e^{2x}$ and sketch.
13. Sketch the curve $y = \frac{1+\ln x}{x}$
14. Evaluate the exact sum of the first 20 terms of $\ln 64 + \ln 32 + \ln 16 + \dots$
15. For what values of θ does $\tan^2 \theta - \tan^4 \theta + \tan^6 \theta - \dots$ have a limiting sum? For these values find the limiting sum in its simplest form.

16. Solve $\frac{1}{x-1} + \frac{1}{x-2} > 1$
17. Evaluate $\int_0^{\frac{1}{2}} u^2 \sqrt{1-u^2} du$ using the substitution $u = \sin \theta$
18. The tangent to the curve $f(x) = ax^2$ at the point $x = 1$ is inclined at an angle of $+\frac{\pi}{4}$ to the line $g(x) = (2 - \sqrt{3})x + b$. Find the values of a and b if $f(1) = g(1)$.
19. Prove by induction that $4 \times 6^{2n} + 3 \times 2^{3n}$ is divisible by 7 for all $n > 0$.
20. If α, β and γ are the roots of $x^3 + x^2 - 22x - 40$, find the polynomial with roots $\alpha + 1, \beta + 1$ and $\gamma + 1$.
21. When $P(x) = 2x^3 + ax^2 + bx + 3$ is divided by $(x + 2)$ the remainder is -39 and when $P'(x)$ is divided by $(x - 3)$ the remainder is 43. Find a and b .
22. Consider the parabola $x^2 = 4ay$ with points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ with $OP \perp OQ$ where O is the origin.
- Find a relationship between p and q .
 - Find the coordinates of M , the midpoint of PQ .
 - Find the coordinates of R such that $POQR$ is a rectangle.
 - Find the locus of R .
23. Consider the parabola $x^2 = 4ay$ with point $P(2ap, ap^2)$. Find the coordinates of:
- the y -intercept, T , of the tangent at P .
 - the midpoint, M , of the interval between P and the focus.
 - the midpoint, R , of MT and find the locus of R .
24. Prove by induction that

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$