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**HSC Extension 2 Mathematics Revision Questions - Set 2**

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1. Let $w = \frac{2-z}{z}$. Find the locus of $w$ when $z$ is a point that moves around the circle with centre the origin and radius 3.

2. Let $S(ae, 0)$ and $S'(-ae, 0)$ be the foci on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $P(x, y)$ be a point on the ellipse. Prove that the angle between the tangent at $P$ and the line $PS$ equals the angle between the tangent and the line $PS'$.

3. A resistance equal to $v + v^3$ is applied to a particle of unit mass moving in a straight line, where $v$ is the velocity. The particle is initially at the origin and moving with velocity $u > 0$.
   (i) Find an expression for the displacement in terms of $v$.
   (ii) Find an expression for time $t$ in terms of $v$.
   (iii) Find an expression for $v$ in terms of $t$.
   (iv) Describe the motion of the particle.

4. The area bounded by $y = \frac{1}{x}$ and $y = \frac{1}{x-1}$ and the lines $y = 1$ and $y = 4$ is rotated around the $y$-axis. Find the volume of the solid so formed, using:
   (i) the method of slicing
   (ii) cylindrical shells
   (iii) the standard approach using $V = \pi \int x^2 \mathrm{d}y$

5. Given that $\alpha, \beta$ and $\gamma$ are the roots of $x^3+ax^2+bx+c = 0$, find the equation that has roots $\alpha^2, \beta^2$ and $\gamma^2$. Hence find $\alpha^2 + \beta^2 + \gamma^2$. Compare your result using the usual method starting with $(\alpha + \beta + \gamma)^2$.

6. A simple pendulum consists of a particle $P$ of mass $m$ kg attached to a fixed point $F$ by a string of length $l$ metres. The particle moves along a circular arc in a fixed vertical plane through $F$ with the point $O$ being the lowest point of the arc. Let $\angle OFP = \theta$, arc length $OP = s$ metres, time $t$ measured in seconds and $g$ $m/s^2$ be gravitational acceleration.
   (i) Show that the tangential acceleration of $P$ be given by

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = l \frac{\mathrm{d}}{\mathrm{d}\theta}(\frac{1}{2}\dot{\theta}^2)$$

where $\dot{\theta} = \frac{\mathrm{d}\theta}{\mathrm{d}t}$.
(ii) Show that the equation of motion of the pendulum is

$$l\frac{\mathrm{d}}{\mathrm{d}\theta}(\frac{1}{2}\dot{\theta}^2) = -g\sin\theta$$

(iii) Suppose the pendulum is given an initial angular velocity of $\sqrt{\frac{g}{l}}$ radians/sec at $\theta = 0$. Show that

$$\frac{1}{2}l\dot{\theta}^2 = g(\cos\theta - \frac{1}{2})$$

Hence deduce that the maximum value of $\theta$ attained by the pendulum is $\frac{\pi}{3}$.
(iv) Suppose that on the initial upward swing the angular velocity is better approximated from the equation

$$\frac{1}{2}l\dot{\theta}^2 = g(\cos\theta - \frac{1}{2}) - \frac{g}{10}(2\sin\theta - \theta).$$

Use one application of Newton's method with $\theta_0 = \frac{\pi}{3}$ to find the maximum value of $\theta$ attained by the pendulum.

7. Let $\alpha, \beta, \gamma$ be the roots of $x^3 + ax^2 + bx + c = 0$. If $b - a - c = 7$ show that $\alpha\beta\gamma < 1$.

8. The probability that an manufacturing company experiences $n$ industrial accidents in one month is given by $P_n = \mathrm{e}^{-1.5}\frac{1.5^n}{n!}$ where $n$ is an integer.
(i) Find the probability that no accidents occur during a particular month.
(ii) Find the probability that there are at least three months in a given year that no accidents occur.
(iii) Find the most likely number of accidents per month.

9. Let $s_n = 1 + \sum_{r=1}^{n} \frac{1}{r!}$ for $n = 1, 2, 3, ....$
(i) Prove using induction that

$$\mathrm{e} - s_n = \mathrm{e}\int_0^1 \frac{x^n}{n!}\mathrm{e}^{-x}\mathrm{d}x$$

(ii) Hence deduce that for $n = 1, 2, 3, ....$

$$0 < \mathrm{e} - s_n < \frac{3}{(n+1)!}$$

(iii) Hence deduce that $(\mathrm{e} - s_n)n!$ is not an integer for $n = 1, 2, 3, ...$
(iv) Show that there cannot exist positive integers $p$ and $q$ such that $\mathrm{e} = \frac{p}{q}$.

10. By considering the exact area under the curve $f(x) = \ln\, x$ between $x = 1$ and $x = k$ with the estimate using the trapezoidal rule, show that

$$k! < \mathrm{e}\sqrt{k}(\frac{k}{\mathrm{e}})^k$$