KENDERDINE
MATHS TUTORING

ABN 49 278 719 954

6 Braeside Dr
BOWRAL NSW 2576
AUSTRALIA

4862 2827 mob 0429 317 491
rdkdine@acenet.com.au
www.kenderdinemathstutoring.com.au

HSC Extension 1 Mathematics Revision Questions - Set 2

Prepared by Dr Richard D Kenderdine

1. The results of a survey showed that 20% of lightglobes had failed before 1000 hours of use. If ten identical lightglobes are installed in a new house, what is the probability that exactly 4 will have failed before 1000 hours of use?

2. How many arrangements of the letters of the word TALLANGATTA are possible?

3. A committee of 5 people has to be chosen randomly from 8 Australians and 6 New Zealanders.
(i) How many committees can be formed if there must be a majority of Australians?
(ii) How many committees can be formed if the chairman must be a New Zealander?
(iii) Suppose the chairman is chosen first, then the remainder of the committee. What is the probability that the chairman is an Australian and the committee, including the chairman, has a majority of New Zealanders?

4. A conical container with base angle $\frac{\pi}{3}$ is being filled with water at the rate of 5 litres/sec. If the radius of the base is r metres and the height h metres, find the height at which the rate of change in depth of water in the container is 1.19 metres/sec.

5. A drink is removed from a cooler at 8° and warms to 16° after 30 mins. If the drink is assumed to warm at a rate proportional to the difference between its temperature and the temperature of the surrounding air, which is at 25°, find

   (i) its temperaure after 1 hour

   (ii) how long it took to reach 20°.

6. The population growth curve which often arises in biology is satisfies

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{k(C - N(t))}{C}$$

where $N(t)$ is the population at time t, k is the rate of growth and C is the maximum sustainable population (carrying capacity).

(i) Derive the equation for $N(t)$

(ii) If the initial population is 1000, the carrying capacity is 2000 and it took 10 years to reach a population of 1200, find
(α) the population after 50 years, and
(β) how long it will take to have a population of 1800.

7. Consider the function

$$N(t) = \frac{100}{1 + b\mathrm{e}^{-kt}}$$

(i) Find values for b and k given that $N(0) = 20$ and $N(20) = 50$

(ii) Find t such that $N(t) = 60$

(iii) Show that $\frac{\mathrm{d}N}{\mathrm{d}t} = AN(C - N)$ where A and C are constant and specify the values of the constants.

8. Starting with $\ddot{x} = \frac{\mathrm{d}v}{\mathrm{d}t}$, prove $\ddot{x} = \frac{\mathrm{d}}{\mathrm{d}x}(\frac{1}{2}v^2)$.

9. A particle moves in a straight line according to $\ddot{x} = \frac{2}{\sqrt{x}}$. Initially it is at $x = 1\ cm$ and moving with velocity $2\sqrt{2}\ cm/s$.

(i) Find the equation for velocity in terms of displacement.
(ii) Find the equation for displacement in terms of time.
(iii) Describe the motion of the particle.

10. A particle moves in a straight line according to $\ddot{x} = -k\dot{x}^2$ where k is a positive constant and $\dot{x} > 0$. If the initial speed is $V\ cm/s$ at $x = 0\ cm$ show that

(i) $\dot{x} = \frac{V}{1+kVt}$
(ii) $x = \frac{1}{k}\ln(1 + kVt)$
(iii) $\dot{x} = V\mathrm{e}^{-kx}$

11. A particle moves in a straight line from its initial position $x = 1\ cm$ with velocity $v = \frac{9-x^2}{x}$

(i) Find an expression for the particle's acceleration in terms of displacement.
(ii) Find an expression for time in terms of displacement and hence an expression for displacement in terms of time.
(iii) Sketch a graph of displacement against time.
(iv) Find the time to travel from $x = 1\ cm$ to $x = 2\ cm$.

12. A particle is moving in a straight line from its initial position $x = \frac{\pi}{2}\ cm$ with velocity given by $v = -\sin^2 x$.

(i) Find an expression for acceleration in terms of displacement.
(ii) Find an expression for displacement in terms of time.
(iii) Sketch a graph of displacement against time.

13. A particle moves in a straight line with position at time t given by

$$x = 1 + \sin 4t + \sqrt{3}\cos 4t$$

(i) Prove that the particle is undergoing SHM with centre $x = 1$
(ii) Find the amplitude

(iii) When does the particle first reach maximum speed after $t = 0$?
(1998 HSC)

14. A particle moves in a straight line with position at time t given by

$$x = \sin^2 2t$$

(i) When is the particle first at $x = \frac{1}{2}$?

(ii) In which direction is the particle travelling at $x = \frac{1}{2}$?

(iii) Find the acceleration in terms of displacement.

(iv) State whether the particle is undergoing SHM.

(v) Determine the period of motion.

15. A particle moves in a straight line under SHM. Initially it is 3 cm to the right of the origin and travelling with a velocity of 2 cm/sec. Find the maximum velocity and acceleration if the period is 8 sec.

16. A projectile is thrown from a 20m high vertical cliff with a velocity of 15m/s at an angle of elevation of 20°.

(i) Derive the equations for velocity and position in terms of time.

(ii) Calculate the distance of the point of impact from the base of the cliff and the time of flight.

(iii) Calculate the maximum height of the projectile.

(iv) Calculate the angle between the path of the projectile and the ground at the point of impact.

(v) Determine the cartesian form of the path of the projectile.

17. A particle is projected from the origin with a velocity of 40 m/s and angle of elevation θ. Assuming $g = 10\ m/s^2$:

(i) Determine the cartesian equation of the path of the projectile.

(ii) Determine the condition for there to be two values of θ in the range $(0, \frac{\pi}{2})$ for the projectile to pass through a given point P with coordinates (X, Y).

(iii) If P also lies on the line $y = x$ and α and β are the two values of θ, find the value of $\alpha + \beta$.