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HSC Extension 1 Mathematics Revision Questions - Set 1

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1. Find $\int \sin^2 5x \, dx$
2. Solve $\frac{2}{x-3} < 4$ and $\frac{x+3}{x-3} < 4$
3. Find the acute angle between the lines $y = 5x - 1$ and $y = 2x + 1$.
4. The angle between two lines is 1.05 radians. If the equation of one line is $y = -2x + 6$, find the gradient of the other.
5. Prove by induction:
 - (a) $2 + 6 + 10 + \dots + (4n - 2) = 2n^2$ for $n \geq 1$
 - (b) $7^n - 2^n$ is divisible by 5 for $n \geq 1$
 - (c) $n^3 \geq 5n - 4$ for $n \geq 1$
6. $(x - 2)$ is a factor of the polynomial $P(x)$ and the remainder after dividing by $(x + 2)$ is -12. When $P(x)$ is divided by $(x - 2)(x + 2)$ the remainder is $R(x)$.
 - (i) Explain why the general form of $R(x)$ is $R(x) = ax + b$
 - (ii) Find values of a and b
7. It is known that one of the roots of $x^3 - 9x^2 + 20x + k = 0$ is twice the sum of the other two roots. Find the value of k .
8.
 - (i) Find the domain for which $f(x) = x^2 + 4x - 1$ is monotonic increasing.
 - (ii) Find and sketch the inverse function, $f^{-1}(x)$, for this domain.
9. Given $f(x) = \frac{x}{x+1}$ for $x > 1$, find the derivative of the inverse function by

(i) using the relationship $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$, given $y = f(x)$ and $x = f^{-1}(y)$.

(ii) finding the inverse function $x = f^{-1}(y)$, finding $\frac{dx}{dy}$ and expressing in terms of x .

10. Find the domain and range of:

(i) $2 \sin^{-1}(1-x)$ (ii) $\frac{1}{3} \cos^{-1} \sqrt{x}$ (iii) $4 \tan^{-1} 3x$

(iv) $-3 \sin^{-1}(\ln x)$ (v) $-\frac{1}{2} \cos^{-1}(4x+1)$

11. Differentiate:

(i) $\tan^{-1} \frac{1}{x}$ (ii) $2 \cos^{-1}(x^2-1)$ (iii) $\sin^{-1}(\ln x)$

(iv) $4 \cos^{-1}(e^x)$ (v) $x \sin^{-1} x$ (vi) $\ln(\sin^{-1} x)$

(vii) $\sin^{-1}(\cos x)$ (viii) $\frac{\tan^{-1} \sqrt{x}}{x}$

12. Evaluate:

(i) $\int_0^1 \frac{1}{\sqrt{2-x^2}} dx$

(ii) $\int \frac{1}{x^2+49} dx$

(iii) $\int_{-1}^1 \frac{1}{\sqrt{4-x^2}} dx$

(iv) $\int_0^{\sqrt{3}} \frac{4}{x^2+9} dx$

13. Find the area enclosed by the function $f(x) = \frac{1}{\sqrt{4-x^2}}$, the x -axis and the lines $x = -1$ and $x = 1$.

14. Find the volume of the solid of revolution formed when the function $y = \frac{1}{\sqrt{4+x^2}}$ is rotated around the x -axis from $x = 0$ to $x = 2$.

15. Find the area bounded by the curve $y = \sin^{-1} x$, the x -axis and the line $x = \frac{1}{2}$.

16. Prove $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ in two different ways.

17. Find the coefficient of x^{-3} in $(1 + \frac{1}{x})^5(1 - \frac{1}{x})^7$

18. Find the largest term (in unsimplified form) in the expansion of $(7 + 5x)^{13}$

19. By writing $(1+x)^n$ as $(1+x)^m(1+x)^{n-m}$ show that

$$\sum_{j=0}^m {}^m C_j {}^{n-m} C_{k-j} = {}^n C_k$$

20. Show that ${}^n C_0 + 2 {}^n C_1 + 3 {}^n C_2 + \dots + (n+1) {}^n C_n = (n+2)2^{n-1}$